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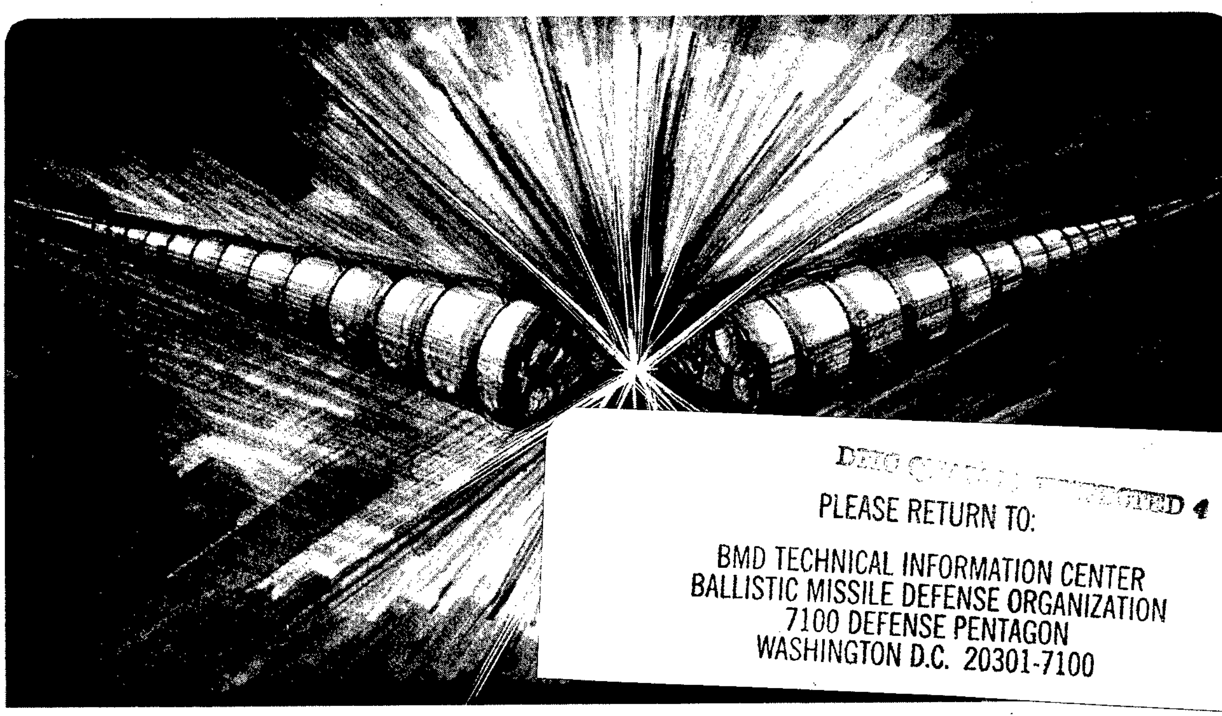
COMPARISON OF ELECTRIC AND MAGNETIC SELF FORCES IN A TOROIDAL GEOMETRY

L. Jackson Laslett

October 1981



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Lawrence Berkeley Laboratory

Technical Report of the Betatron Design Study

COMPARISON OF ELECTRIC AND MAGNETIC SELF FORCES IN A TOROIDAL GEOMETRY*

L. Jackson Laslett

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October 18, 1981

*Sponsored by Defense Advance Research Projects Agency (DoD)
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Magnetic and electric self fields are evaluated for a rectangular beam centered within a closed toroidal chamber of rectangular cross section. The chamber walls are regarded as conductors, at zero electrostatic potential, that exclude the time varying magnetic fields of the beam. Expressions are derived for the axial derivatives of the electric and magnetic fields in the mid-plane, and numerical evaluations of these derivatives are reported for examples relevant to betatron design. Attention is directed to the failure

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20. ABSTRACT (Continued)

of the electric and magnetic self forces to exhibit, in this toroidal geometry, the strong $1 - \beta^2$ cancellation characteristic of the self fields of a beam in a straight conducting chamber.

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COMPARISON OF ELECTRIC AND MAGNETIC SELF FORCES IN A TOROIDAL GEOMETRY*

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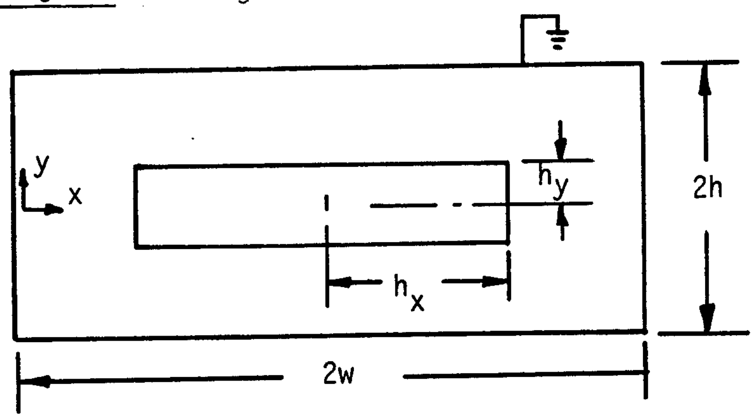
LINEAR GEOMETRY

We consider first the electrostatic problem of a rectangular beam in a straight rectangular box (cgs, esu).



Green's Function,

for line charge $\rho\delta(x - x_1)\ \delta(y - y_1)$ at x_1, y_1:

$y > y_1$:

$$V_> = 8\rho \sum_n \frac{\text{Sinh}\,\frac{n\pi(h + y_1)}{2\,w}}{\text{Sinh}\,\frac{n\pi h}{w}} \sin\frac{n\pi x_1}{2w} \sin\frac{n\pi x}{2w} \,\text{Sinh}\,\frac{n\pi(h-y)}{2w}$$

$y < y_1$:

$$V_< = 8\rho \sum_n \frac{\text{Sinh}\,\frac{n\pi(h - y_1)}{2\,w}}{\text{Sinh}\,\frac{n\pi h}{w}} \sin\frac{n\pi x_1}{2w} \sin\frac{n\pi x}{2w} \,\text{Sinh}\,\frac{n\pi(h+y)}{2w} \quad .$$

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By integration over x_1 from $w - h_x$ to $w + h_x$, and by integration over y_1 from $-h_y$ to y using $V_>$ and from y to h_y using $V_<$, the potential function within the (uniform) beam becomes

$$V = \frac{64\ \rho\ w^2}{\pi^2} \sum_n \frac{1}{n^3} \left[1 - \frac{\operatorname{Cosh} \dfrac{n\pi\ (h-h_y)}{2w} \operatorname{Cosh} \dfrac{n\pi y}{2w}}{\operatorname{Cosh} \dfrac{n\pi h}{2w}} \right] \sin \frac{n\pi}{2} \quad \sin \frac{n\pi h_x}{2w} \sin \frac{n\pi x}{2w} ,$$

wherein it will be recognized that $\sin \frac{n\pi}{2}$ is equal to

	+1	0	-1	0	+1	etc.
for n =	1	2	3	4	5	etc.

As a check, we note that

$$\nabla^2 V = -16\ \rho \sum_n \frac{1}{n} \sin \frac{n\pi}{2} \quad \sin \frac{n\pi h_x}{2w} \sin \frac{n\pi x}{2w}$$

$$= \begin{cases} -4\pi\rho & \text{for } w - h_x < x < w + h_x \\ 0 & \text{for } 0 < x < w - h_x \text{ and for } w + h_x < x < 2w. \end{cases}$$

[In particular, if $h_x = w$,

$$\nabla^2 V = -16\ \rho \left[\sin \frac{\pi x}{2w} + \frac{1}{3} \sin 3 \frac{\pi x}{2w} + \frac{1}{5} \sin 5 \frac{\pi x}{2w} + \ldots \right]$$

$$= -16\ \rho\ \frac{\pi}{4} = -4\pi\rho \text{ (for } 0 < x < 2w). \qquad \text{[cf. B.O. Pierce, Eq. 808.]}$$
]

If we form $\left. \dfrac{\partial(-\frac{\partial V}{\partial y})}{\partial y} \right|_{y=0}$,

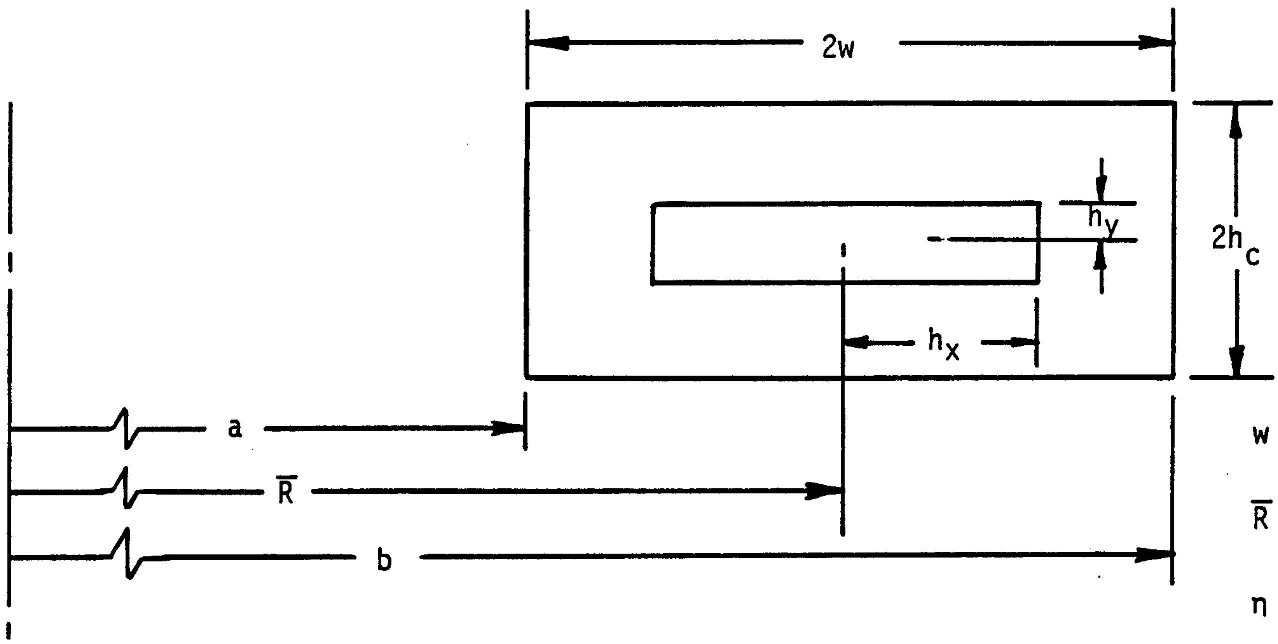
we obtain

$$16\ \rho \sum_n \frac{1}{n} \frac{\operatorname{Cosh} \dfrac{n\pi(h - h_y)}{2w}}{\operatorname{Cosh} \dfrac{n\pi h}{2w}} \quad \sin \frac{n\pi}{2} \sin \frac{n\pi h_x}{2w} \quad \sin \frac{n\pi x}{2w} \quad . \tag{1}$$

When self magnetic forces are also taken into account in this straight-pipe situation, correspondingly the gradient of effective field becomes multiplied by the factor $1 - \beta^2$, where $\beta = v/c$.

TOROIDAL GEOMETRY

We next consider the corresponding problem in toroidal geometry.



$$w = \frac{b-a}{2}$$

$$\overline{R} = \frac{b+a}{2}$$

$$\eta = \frac{w}{\overline{R}} = \frac{b-a}{b+a}$$

This toroidal case has been analysed by Dr. Lloyd Smith. For the electrostatic problem we now employ, in our present notation, radial functions

$$F_0(n;r) = \alpha_0 \left[J_0(k_n r)\, Y_0(k_n a) - Y_0(k_n r)\, J_0(k_n a) \right]$$

$$= \alpha_0 \left[J_0\left(z_0(n)\frac{r}{\overline{R}}\right) Y_0\left(z_0(n)\frac{a}{\overline{R}}\right) - Y_0\left(z_0(n)\frac{r}{\overline{R}}\right) J_0\left(z_0(n)\frac{a}{\overline{R}}\right) \right] ,$$

where the k_n, or $z_0(n) = k_n \overline{R}$, are such as to make $F_0(n;r)$ vanish for $r = b$ and α_0 is an arbitrarily selected factor (e.g., $\alpha_0 = 1$).

The Green's function then may be written:

$y > y_1$:

$$V_> = 4\pi\rho\overline{R} \sum_n \frac{r_1\, F_0(n;r_1)\, F_0(n;r)}{z_0(n) \int_a^b r' [F_0(n;r')]^2\, dr'} \; \frac{\operatorname{Sinh} z_0(n) \dfrac{h_c + y_1}{\overline{R}} \operatorname{Sinh} z_0(n) \dfrac{h_c - y}{\overline{R}}}{\operatorname{Sinh} 2z_0 \dfrac{h_c}{\overline{R}}}$$

$y < y_1$:

$$V_< = 4\pi\rho\bar{R} \sum_n \frac{r_1\, F_0(n;r_1)\, F_0(n;r)}{z_0(n)\int_a^b r'[F_0(n;r')]^2\, dr'} \; \frac{\text{Sinh } z_0(n)\,\frac{h_c - y_1}{\bar{R}}\; \text{Sinh } z_0(n)\,\frac{h_c + y}{\bar{R}}}{\text{Sinh } 2z_0\,\frac{h_c}{\bar{R}}}$$

Integration over the location r_1, y_1, of the source involves first the following

$$\frac{\text{Sinh } z_0(n)\frac{h_c - y}{\bar{R}} \int_{-h_y}^{y} \text{Sinh } z_0(n)\frac{h_c + y_1}{\bar{R}}\, dy_1 + \text{Sinh } z_0(n)\,\frac{h_c + y}{\bar{R}} \int_y^{h_y} \text{Sinh } z_0(n)\,\frac{h_c - y_1}{\bar{R}}}{\text{Sinh } 2\, z_0(n)\,\frac{h_c}{\bar{R}}}$$

$$= \frac{\frac{\bar{R}}{z_0(n)}\left\{\text{Sinh } z_0(n)\,\frac{h_c - y}{\bar{R}}\left[\text{Cosh } z_0(n)\frac{h_c + y}{\bar{R}} - \text{Cosh } z_0(n)\,\frac{h_c - h_y}{\bar{R}}\right] + \text{Sinh } z_0(n)\,\frac{h_c + y}{\bar{R}}\left[\text{Cosh } z_0(n)\,\frac{h_c - y}{\bar{R}} - \text{Cosh } z_0(n)\frac{h_c - h_y}{\bar{R}}\right]\right\}}{\text{Sinh } 2\, z_0(n)\,\frac{h_c}{\bar{R}}}$$

$$= \frac{\bar{R}}{z_0(n)}\left[\frac{\text{Sinh } 2\, z_0(n)\,\frac{h_c}{\bar{R}} - 2\,\text{Cosh } z_0(n)\frac{h_c - h_y}{\bar{R}}\;\text{Sinh } z_0(n)\frac{h_c}{\bar{R}}\;\text{Cosh } z_0(n)\,\frac{y}{\bar{R}}}{\text{Sinh } 2\, z_0(n)\,\frac{h_c}{\bar{R}}}\right]$$

$$= \frac{\bar{R}}{z_0(n)}\left[1 - \frac{\text{Cosh } z_0(n)\,\frac{h_c - h_y}{\bar{R}}}{\text{Cosh } z_0(n)\,\frac{h_c}{\bar{R}}}\;\text{Cosh } z_0(n)\;\frac{y}{\bar{R}}\right],$$

Followed by an integration over r_1 (from $\bar{R} - h_x$ to $\bar{R} + h_x$).

For this latter integration we follow the suggestion of Dr. Smith in that we take $\rho(r_1) = \bar{\rho}\,\dfrac{\bar{R}}{r_1}$, whereupon we obtain

$$V = 4\pi\bar{\rho}\,\bar{R}^3 \sum \frac{\left[\displaystyle\int_{\bar{R}-h_x}^{\bar{R}+h_x} F_0(n;r_1)dr_1\right] F_0(n;r)}{\left[z_0(n)\right]^2 \displaystyle\int_a^b r'\,[F_0(n;r')]^2\,dr'} \quad X$$

$$\left[1 - \frac{\cosh z_0(n)\dfrac{h_c - h_y}{\bar{R}}}{\cosh z_0(n)\,\dfrac{h_c}{\bar{R}}} \cosh z_0(n)\,\frac{y}{\bar{R}}\right] .$$

Finally, then -- for this toroidal electrostatic problem -- if we form

$\left.\dfrac{\partial(-\partial V/\partial y)}{\partial y}\right|_{y=0}$, we obtain

$$4\pi\bar{\rho}\,\bar{R} \sum_n \frac{\displaystyle\int_{\bar{R}-h_x}^{\bar{R}+h_x} F_0(n;r_1)dr_1}{\displaystyle\int_a^b r'\,[F_0(n;r')]^2\,dr'}\; F_0(n;r)\; \frac{\cosh z_0(n)\,\dfrac{h_c - h_y}{\bar{R}}}{\cosh z_0(n)\,\dfrac{h_c}{\bar{R}}} \;. \qquad (2a)$$

Correspondingly, for the <u>magnetic</u> problem, we obtain a similar result -- save that

(i) a function F_1 is employed, in which the Bessel functions are of order 1,

(ii) the zeros are the quantities $z_1(n)$ for which $F_1(n;b) = 0$,

and

(iii) a factor $-\beta^2$ is to be appended for the contribution to the force gradient:

$$4\pi\bar{\rho}\,\bar{R}\,(-\beta^2)\sum_n \frac{\int_{\bar{R}-h_x}^{\bar{R}+h_x} F_1(n;r_1)dr_1}{\int_a^b r'[F_1(n;r')]^2 dr'}\, F_1(n;r)\, \frac{\text{Cosh } z_1(n)\, \frac{h_c - h_y}{\bar{R}}}{\text{Cosh } z_1(n)\, \frac{h_c}{\bar{R}}} \qquad (2b)$$

These terms [(2a) and (2b)] combined give the expression proposed by Dr. Smith, if we identify our $\bar{\rho}\bar{R}$ with his $e\rho_0 R$.

In the work that follows, we shall identify

$$\text{TERM 1} = \bar{R}\sum_n \frac{\int_{\bar{R}-h_x}^{\bar{R}+h_x} F_0(n;r_1)dr_1}{\int_a^b r'[F_0(n;r')]^2 dr'}\, F_0(n;r)\, \frac{\text{Cosh } z_0(n)\, \frac{h_c - h_y}{\bar{R}}}{\text{Cosh } z_0(n)\frac{h_c}{\bar{R}}} \qquad (3a)$$

$$\text{TERM 2} = \bar{R}\sum_n \frac{\int_{\bar{R}-h_x}^{\bar{R}+h_x} F_1(n;r_1)dr_1}{\int_a^b r'[F_1(n;r')]^2 dr'}\, F_1(n;r)\, \frac{\text{Cosh } z_1(n)\, \frac{h_c - h_y}{\bar{R}}}{\text{Cosh } z_1(n)\frac{h_c}{\bar{R}}}\,. \qquad (3b)$$

Following what we believe to have been the suggestion of Dr. Smith, we have undertaken to evaluate the integrals (and the subsequent sums)that appear in the above expressions <u>numerically.</u>*

*4th order integration by use of the extended Simpson's Rule (Abramovitz and Stegun, §25.4.6)

We remark that TERM 1 and TERM 2 will be rather close to one another numerically -- and indeed, when w/R << 1, will each be rather close* to the quantity

$$\frac{4}{\pi}\sum_{n}\frac{1}{n}\frac{\operatorname{Cosh}\frac{n\pi(h-h_y)}{2w}}{\operatorname{Cosh}\frac{n\pi h}{2w}}\sin\frac{n\pi}{2}\sin\frac{n\pi h_x}{2w}\sin\frac{n\pi x}{2w} \tag{4}$$

exhibited by expression (1) [p. 2] divided by $4\pi\rho$.

---

*Thus if F_0 (or F_1) be approximated by

$$F_0 \cong F_1 \cong \sin\left[n\left(1+\frac{r-\bar{R}}{\left(\frac{b-a}{2}\right)}\right)\frac{\pi}{2}\right]$$

$$= \sin\left[n\left(1+\frac{r-\bar{R}}{w}\right)\frac{\pi}{2}\right] \quad , \text{ with } \frac{w}{\bar{R}} << 1,$$

and $z_0(n) \cong z_1(n) \cong n\frac{\bar{R}}{w}\frac{\pi}{2}$,

$$\text{TERM 1} \cong \text{TERM 2} \cong \bar{R}\sum_{n}\frac{\frac{2w}{n\pi}\left[\cos n(1-\frac{h_x}{w})\frac{\pi}{2} - \cos n\ (1+\frac{h_x}{w})\ \frac{\pi}{2}\right]}{\bar{R}\ (2w)\ \frac{1}{2}}$$

$$\times\ \sin\left[n(1+\frac{r-\bar{R}}{w})\ \frac{\pi}{2}\right]\frac{\operatorname{Cosh}\frac{n\pi(h_c-h_y)}{2w}}{\operatorname{Cosh}\frac{n\pi h_c}{2w}}$$

$$= \frac{4}{\pi}\sum_{n}\frac{1}{n}\frac{\operatorname{Cosh}\frac{n\pi(h_c-h_y)}{2w}}{\operatorname{Cosh}\frac{n\pi h_c}{2w}}\sin n\frac{\pi}{2}\sin n\frac{\pi h_x}{2w}\sin\frac{n\pi x}{2w}$$

if we identify $x = r - (\bar{R} - w)$

$= r - a$

-- thus providing agreement, in this limit, with expression (4). Note that terms for which n is <u>even</u> will be expected to be small [since the factor $\sin n\,\pi/2$ in the above approximation vanishes for n even]. Also, moreover, if $h_x = w$ and $x = w(r = \bar{R})$, this approximation suggests that

$$\text{TERM 1} \cong \text{TERM 2} \cong \frac{4}{\pi}\left[\frac{\text{Cosh}\,\dfrac{\pi(h_c - h_y)}{2w}}{\text{Cosh}\,\dfrac{\pi h_c}{2w}} - \frac{1}{3}\,\frac{\text{Cosh}\,\dfrac{3\pi(h_c - h_y)}{2w}}{\text{Cosh}\,\dfrac{3\pi h_c}{2w}} + \frac{1}{5}\,\frac{\text{Cosh}\,\dfrac{5\pi(h_c - h_y)}{2w}}{\text{Cosh}\,\dfrac{5\pi h_c}{2w}} - \ldots\right].$$

Such approximate behavior has proven of use in checking some aspects of operation of the computer program CPRTS.

<u>COMMENT:</u>

We recall that for a straight beam pipe the electric and magnetic self forces are expected virtually to cancel, through the action of the factor $\frac{1}{\gamma^2} = 1 - \beta^2$.

In the toroidal geometry, on the other hand, the electric and magnetic forces may differ slightly in magnitude even in the limit $\beta^2 \gg 1$.

To compare the relative sizes of such effects, it may be helpful, therefore, to think of a quantity analogous to $\frac{1}{\gamma^2}$ that we form from the toroidal solutions as

$$\frac{\text{TERM 1} - \text{TERM 2}}{\text{Avg. of TERM 1 and TERM 2}}$$

-- <u>i</u>.<u>e</u>.,

$$\frac{2}{\text{TERM 1} + \text{TERM 2}} \cdot (\text{TERM 1} - \text{TERM 2}).$$

THE PROGRAM

The program CPRTS (Library LASLETT) first computes the first 100 "zeros" [$z_0(n)$ and $z_1(n)$ of $F_0(n,b)$ and $F_1(n,b)$] by successive application of Newton's method -- employing in this connection the Sub-routine (Mrs. Barbara (Harold) Levine) LJBJY for Bessel functions and (optionally) their 1st derivatives. (The chamber radii, a and b, are entered for this purpose.) Optionally printed, in tabular form, are the $z_0(n)$ and then the $z_1(n)$, each accompanied by the value to which F_0 or F_1 has thereby been reduced to virtually zero.

Next, with h_x entered, the program then computes (and prints)

$$\mathrm{RHO}\phi = \frac{\int_{\bar{R}-h_x}^{\bar{R}+h_x} F_0(n;r_1)dr_1}{\int_a^b r'[F_0(n;r')]^2 dr'} \qquad \mathrm{RHO1} = \frac{\int_{\bar{R}-h_x}^{\bar{R}+h_x} F_1(n;r_1)dr_1}{\int_a^b r'[F_1(n;r')]^2 dr'} .$$

[Integration steps are taken to be inversely proportional to n.] Actually the numerators and denominators of each of these quotients are calculated in two ways: First, directly; second by subtracting some simple form from the integrand and then supplementing the integral of the result by the analytic integral of the simple form that had been subtracted. The ratios (RHOϕ and RHO1) computed by the "direct" and by the "difference" procedure are each printed. One then has the option of selecting whether one will retain for future use results (quotients) obtained by the direct or by the difference procedure and such values are then printed, once again, as a summary. [For a given number of integration steps the results obtained by the difference procedure are believed to be somewhat more accurate.]

By entering h_c, h_y, and the field-point radius (r_B), the program is then in a position to compute the successive terms (n = 1, 2, 3, ...) and to print the successive cumulative sums of the expressions TERM 1 and TERM 2.

Upon the completion of such a tabulation, the program can be directed to accept entries of new values of h_c, h_y, and r_B -- or alternatively to return to earlier stages of the program.

RESULTS

Results for several different parameters (including r_B) are summarized in Tables that follow.

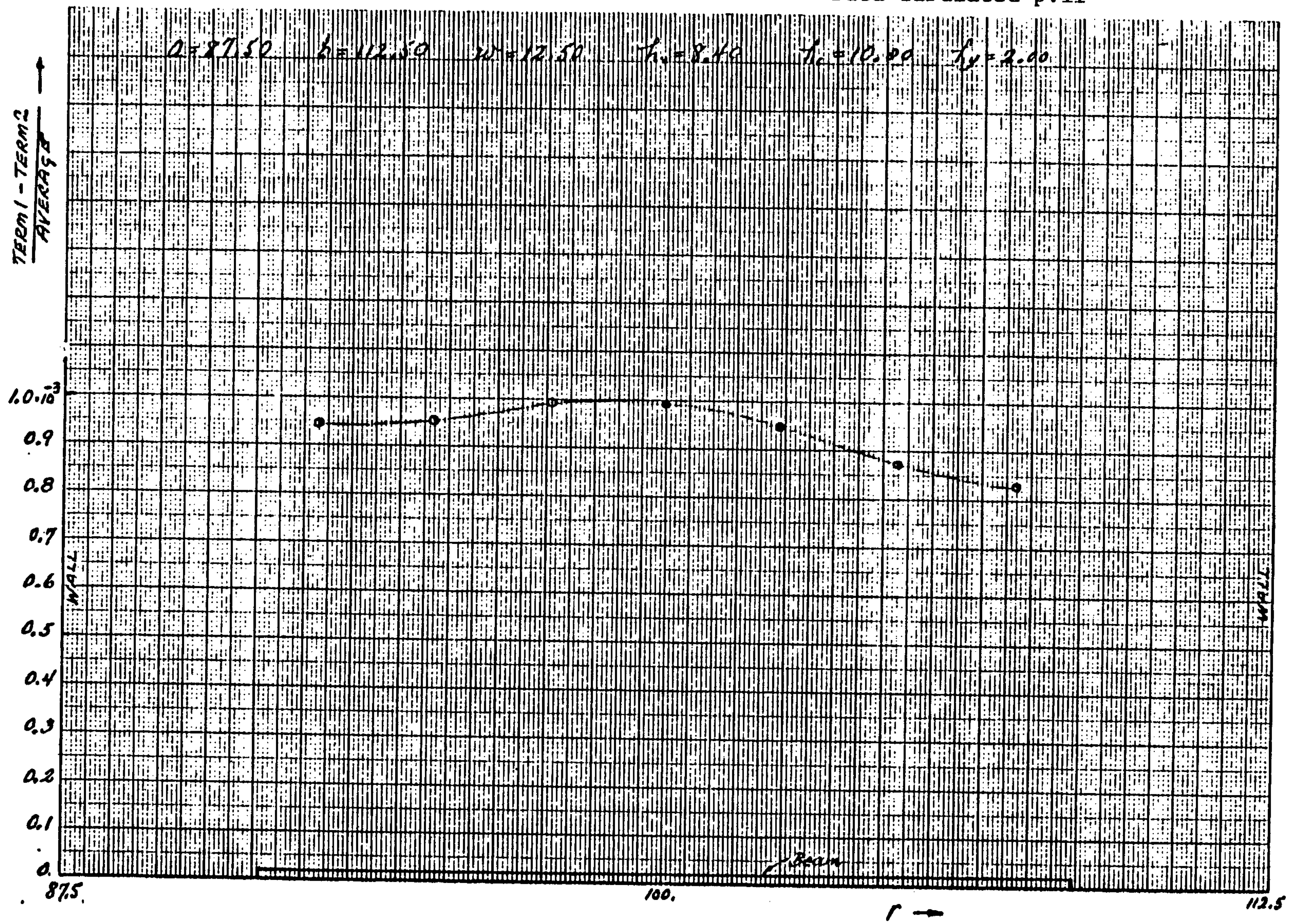
$a = 990.00$ $b = 1010.00$ $w = 10.00$

$h_x = 9.90$

$h_c = 5.00$ $h_y = 4.50$

r_B	TERM 1	TERM 2	$\frac{\text{TERM 1 - TERM 2}}{\text{AVERAGE}}$
1000.00	0.89152 8556	0.89151 9000	1.0719×10^{-5}

Data Tabulated p.11



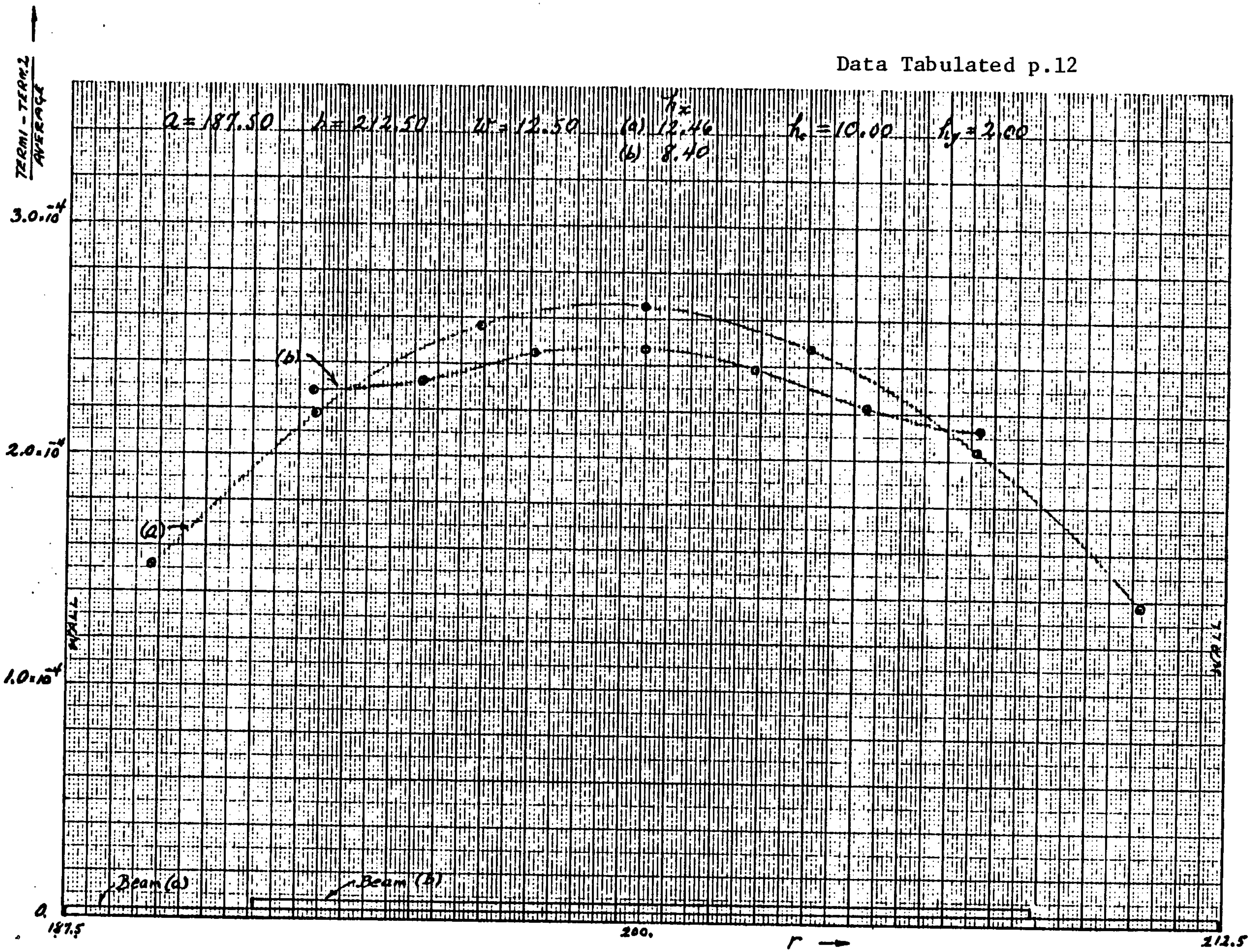
$a = 87.50$ $b = 112.50$ $w = 12.50$

$h_x = 8.40$

$h_c = 10.00$ $h_y = 2.00$

r_B	TERM 1	TERM 2	$\frac{\text{TERM 1 - TERM 2}}{\text{AVERAGE}}$
92.80	0.65204 9938	0.65143 3302	9.46136×10^{-4}
95.20	0.82374 9845	0.82296 2369	9.56422×10^{-4}
97.60	0.86558 6391	0.86472 6339	9.94100×10^{-4}
100.00	0.86185 8622	0.86100 3476	9.92704×10^{-4}
102.40	0.82865 0396	0.82786 4456	9.48908×10^{-4}
104.80	0.75552 2275	0.75486 2845	8.73183×10^{-4}
107.20	0.57508 3712	0.57460 6194	8.30690×10^{-4}

Data Tabulated p.12



$a = 187.50$ $b = 212.50$ $w = 12.50$

$h_x = 12.46$

$h_c = 10.00$ $h_y = 2.00$

r_B	TERM 1	TERM 2	$\frac{\text{TERM 1 - TERM 2}}{\text{AVERAGE}}$
189.32	0.49962 2554	0.49954 6695	1.51844×10^{-4}
192.88	0.81219 1130	0.81201 3801	2.18358×10^{-4}
196.44	0.88358 9404	0.88336 2741	2.56558×10^{-4}
200.00	0.88990 1674	0.88966 5564	2.65357×10^{-4}
203.56	0.85477 4512	0.85456 3070	2.47396×10^{-4}
207.12	0.76014 5318	0.75999 0786	2.03313×10^{-4}
210.68	0.45297 1393	0.45290 9272	1.37151×10^{-4}

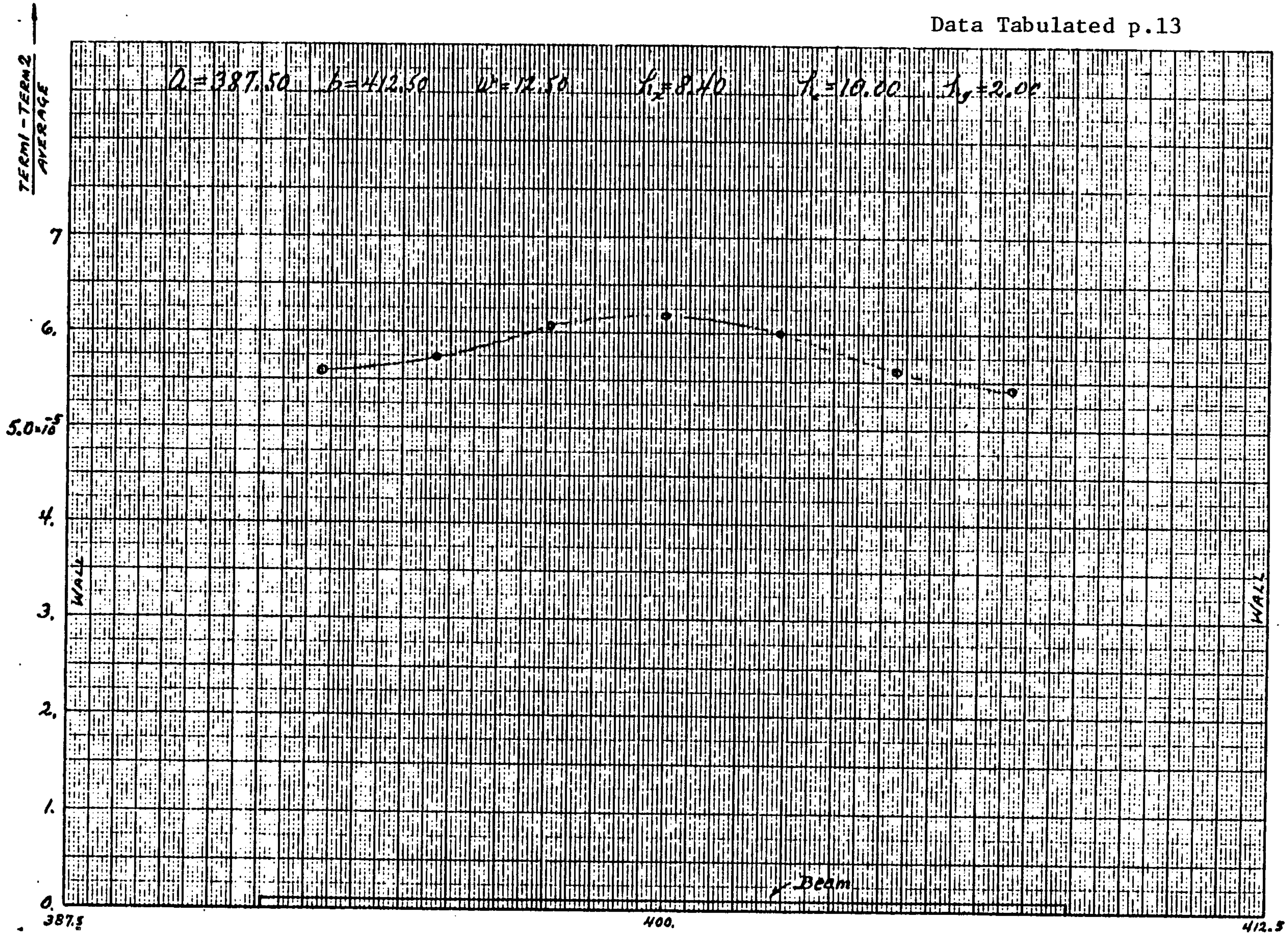
$a = 187.50$ $b = 212.50$ $w = 12.50$ (as above)

$h_x = 8.40$

$h_c = 10.00$ $h_y = 2.00$ (as above)

r_B	TERM 1	TERM 2	$\frac{\text{TERM 1 - TERM 2}}{\text{AVERAGE}}$
192.80	0.63065 1758	0.63050 8072	2.27863×10^{-4}
195.20	0.80521 8479	0.80503 1029	2.32821×10^{-4}
197.60	0.85568 1330	0.85547 1844	2.44848×10^{-4}
200.00	0.86149 2848	0.86127 9728	2.47415×10^{-4}
202.40	0.83723 5836	0.83703 5575	2.39222×10^{-4}
204.80	0.77118 1650	0.77101 0104	2.22470×10^{-4}
207.20	0.59231 7299	0.59219 0831	1.13537×10^{-4}

Data Tabulated p.13



$a = 387.50$ $b = 412.50$ $w = 12.50$

$h_x = 8.40$

$h_c = 10.00$ $h_y = 2.00$

r_B	TERM 1	TERM 2	$\frac{\text{TERM 1 - TERM 2}}{\text{AVERAGE}}$
392.80	0.62054 1001	0.62050 6263	5.5982×10^{-5}
395.20	0.79634 6686	0.79630 0904	5.7492×10^{-5}
397.60	0.85090 4066	0.85085 2326	2.0808×10^{-5}
400.00	0.86140 1722	0.86134 8483	6.1807×10^{-5}
402.40	0.84168 4122	0.84163 3534	6.0104×10^{-5}
404.80	0.77933 7854	0.77929 4057	5.6199×10^{-5}
407.20	0.60139 2252	0.60135 9661	5.4194×10^{-5}